

Effective Methods for Diophantine Problems

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In 1993 Andrew Wiles proved Fermat's Last Theorem, by proving a small part of the Langlands conjectures (all elliptic curves allow a modular parametrization). One knows that knowledge of good upper bounds on the degree of such modular parametrizations has spectacular consequences for the actual solution of Diophantine problems. Very recently there has been important progress in establishing such upper bounds. The traditional "modular method" for Diophantine equations has thus been refined and extended in a significant manner. As a result the class of Diophantine equations for which the set of solutions can be effectively enumerated has become larger, drastically. The aim of our workshop was to invite the international experts that contributed to these recent exciting developments, and to pin down, as well as to transcend, the boundaries of the current techniques.

In order to reach this goal our idea was to combine and integrate different existing methods from various traditions, in order to try and achieve wider applicability. More precisely, we have tried to combine methods from number theory (such as Baker's method, the Runge-Siegel method, the hypergeometric method) with methods from arithmetic geometry (such as the modular method, the effective Shafarevich method, the Chabauty-Kim method). These well-established methods are and have been successfully applied to, among others, S -unit equations, Thue-Mahler equations, equations of Fermat-type, and certain modular curves.

During the workshop the leading experts generously shared their knowledge, and have made their methods more accessible. New collaborations have emerged, and we now have a good idea where we are, and what are the issues for the near future. In order to catalyze the search for the boundaries of our current knowledge we organized several so-called "what's next" sessions. In these sessions, the leading experts, often young researchers, explained in an informal manner the problems they are currently stuck on.

Among others, the following timely topics were discussed: quadratic Chabauty, bounds for differential forms on Shimura and modular curves, bounds on the Manin constant, effective Shafarevich conjecture for cyclic covers, and the search for "nice" diophantine equations describing moduli stacks of abelian varieties to which the effective Shafarevich method can be successfully applied. During the week, the participants collaborated, and worked hard, on these problems. A final presentation of the (preliminary) results took place on the Thursday afternoon.

The idea of organizing these "what's next" sessions was based on a suggestion of the Lorentz Center. This part of the program was certainly unusual given the structure of the more traditional mathematics conference, which would just be a long sequence of research talks. At the beginning certain participants hesitated whether these sessions would be a good idea, but in the end all believed that the idea worked out very well. The sessions had an open and

informal atmosphere, they lasted longer than planned, and both session leaders and participants dared to speculate and to ask questions. We thank the Lorentz Center for the suggestion and we would certainly advise other workshop organizers to incorporate such sessions in their program.

Looking back on our workshop we have seen a good mix of new techniques from arithmetic geometry, and more traditional methods from number theory. Roughly speaking these are two communities, that both in the end deal with the same type of problems, and we are sure that there has been an interesting cross-fertilization between the two communities. We expect that new outcomes will appear from the exchange of ideas that has taken place during the workshop week, but unfortunately not on a very short term.

Many young researchers participated in the workshop, and gave talks in the afternoon or even in the central “what’s next” sessions. Four established researchers gave overview talks accessible to a larger audience on the last day of the workshop, which was organized together with the Intercity Number Theory seminar. This day drew a lot of extra attention from number theorists from all over The Netherlands. We believe that throughout the week we have created a good balance between younger and more senior researchers.

As a final remark, we saw that during the week co-authors found time to sit together and finalize some of their projects. One of the workshop participants mentioned to us explicitly, after listing the various projects he had been working on with others during the week: “I can confidently say without a doubt that none of these items would have happened had it not been for my attendance at your conference.”

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