Fractality and Fractionality

17–20 May 2016
Lorentz Center, Leiden, The Netherlands

Workshop on Fractality and Fractionality

The self-similarity phenomena, or fractal phenomena, are being actively investigated by various research groups throughout the world. The reason of such profound interest in these phenomena is their ubiquity in different areas. The fractal behavior can appear statically, in which case it is usually referred to as fractality, or dynamically, in which case it is called fractionality. Statically, fractals appear both in natural sciences, such as geophysics, crystallography, astronomy, biology, chemistry, bioinformatics and in different branches of mathematics: number theory, geometry, theory of differential equations etc. Dynamically, fractal behavior is demonstrated by macroscopic collections of the units that are endowed with the potential to evolve in time. Such collections are object of study in fluid mechanics, physics of nano-particles, electronics, cellular communications, economics, financial mathematics and many other areas. Because of its static nature, the word "fractality" is more common when speaking of deterministic objects, while "fractionality" often means stochastic behavior. Modern concepts of multifractality and multifractionality are further extensions of these notions. They are used to describe the phenomena which are only locally self-similar. Again, the locality here may refer to time (fractionality over small time intervals) or to space (fractality in small space domains).

The proposed conference, while being relatively small in size, aims at bringing together the leading specialists of several fields of expertise:

- fractional calculus;
- fractional equations and fractional dynamics;
- fractional stochastic analysis;
- fractional and multifractional stochastic processes;
- applications of fractal and fractional analysis.

The choice of topics was motivated by two factors. Firstly, the organizers aim at the widest coverage possible. Secondly, the topics should not be too distant from each other to allow effective communication. The goals of the workshop are thus:

- to stimulate transfer of scientific ideas between different research areas;
- to start new collaborations between researchers from different countries and different research communities;
- to promote exchange of ideas of practical applications.

Organizers

The workshop is organized by:

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Lorentz Center

Lorentz Center is an international center that coordinates and hosts workshops in the sciences, based on the philosophy that science thrives on interaction between creative researchers. Lorentz Center workshops focus on new collaborations and interactions between scientists from different countries and fields, and with varying seniority. Additional information on Lorentz Center can be found on its webpage at http://www.lorentzcenter.nl.

Venue and directions

Lectures will be held at Lorentz Center@Oort that is located on the campus of Leiden University at Niels Bohrweg 2 and occupies the 3rd floor of the Oort Building there.

Detailed directions for getting from the central Leiden railway station to Lorentz Center are available at
http://www.lorentzcenter.nl/howto.php.
Direction for getting to the center from Hotel Van der Valk can be found at
http://www.lorentzcenter.nl/hotels.php.
The campus map can be accessed at
http://media.leidenuniv.nl/legacy/stafkaart-universiteit-leiden-groot.pdf.

Webpage

For the latest information on the workshop see
http://lorentzcenter.nl/lc/web/2016/779/info.php3?wsid=779&venue=Oort.

Sponsors

The workshop is financially supported by:

- Lorentz Center;
- The Dutch Mathematics Cluster STAR (‘Stochastics – Theoretical and Applied Research’);
- The Netherlands Organization for Scientific Research (NWO);
- Foundation Compositio Mathematica;
- Korteweg-de Vries Institute for Mathematics, University of Amsterdam;
- Radboud University;
- Springer.

Practicalities

Lunches

The lunches are organized in the canteen located on the first floor of Snellius Building. Snellius Building is a building opposite Oort Building.

Conference dinner

The conference dinner is scheduled on Wednesday May 18 and will be combined with a boat trip in the haven and along the canals of Leiden. A bus to the haven will be departing at 16:00 from the Oort Building.

Tourist information

Tourist information on Leiden can be obtained through `http://portal.leiden.nl/en/homepage`.

Catering in Leiden

There is a large number of restaurants and bars downtown Leiden. For additional information and reviews one can consult e.g. `http://www.tripadvisor.com`. A comprehensive source in Dutch is `http://www.iens.nl`.

Questions

For questions of practical nature you can drop by the office of the workshop coordinator Tara Seeger at Lorentz Center.

Announcements concerning after-workshop publications

During the Workshop “Fractality and Fractionality“ or shortly afterwards, some selected unpublished papers will be invited for publication in the specialized SCI indexed journal “Fractional Calculus and Applied Analysis” (FCAA) currently published by De Gruyter. Due to the very great backlog of the journal, only a small portion of papers can be selected among the most close to FCAA’s primary topics or revealing the connection between the two “F”s in the workshop’s title. Details on the journal, including Instructions, Authors Kit, etc. as well as the recent years contents (since 2011) can be found at `http://www.degruyter.com/view/j/fca`, while older stuff (1998-2010) is on `http://www.math.bas.bg/~fcaa` . Research papers are usually up to 12-16 pages, while Survey papers are encouraged and can be longer, please avoid many co-authors, and better present your own results. The deadline for submissions will be specified during the Workshop, but is expected to be by the end of August 2016. All contacts and queries concerning FCAA journal should be sent to Editor-in-Chief, Virginia Kiryakova at `virginia@diogenes.bg` (`fcaa@math.bas.bg`).

The papers of the participants of the Workshop, related to their talks, can also be submitted to the journal "Modern Stochastics: Theory and Applications", which planned to publish (a) special issue(s) devoted to the Fractality and Fractionality Workshop topics. The deadline for submissions will be specified during the Workshop, it is expected to be in the period July–August 2016. Potential authors are already asked to submit their name and title of the paper to the address `yumishura1@gmail.com` no later than June 20, which is one month after the Workshop. Information concerning the journal and guidelines for the submission can be found on `https://www.i-journals.org/vtxpp/VMSTA`.

Schedule

Tuesday, May 17

09:00–10:00 **Reception, registration**

10:00–10:15 **Welcome by the Lorentz Center**

10:15–10:30 **Introduction by the workshop organizers**

10:30–11:10 Jacques Lévy Véhel. *Processes with varying local regularities*

11:10–11:50 Antoine Ayache. *Stationary increments harmonizable stable fields: upper estimates on path behavior*

11:50–12:30 Yurii Kondratiev. *Fractional stochastic dynamics in the continuum*

12:30–14:00 **Lunch**

14:00–14:40 Vassili Kolokoltsov. *Probabilistic approach for solving fractional differential equations of Caputo type and their various extensions*

14:40–15:10 Serge Cohen. *Fractional fields parameterized by manifolds*

15:10–15:40 Joachim Lebovits. *Stochastic calculus with respect to Gaussian processes*

15:40–16:10 Mounir Zili. *Mixed sub-fractional Brownian motion*

16:10–16:30 **Coffee break**

16:30–17:30 **Discussion in groups**

Fractional stochastic calculus

Luisa Beghin
Ehsan Azmoodeh
Lauri Viitasaari

Fractals and stochasticity

Uta Freiberg
Mirko D'Ovidio
Michael Hinz

17:30–20:00 **Wine and Cheese party**

Wednesday, May 18

09:00–09:40	Takashi Kumagai. *Time changes of stochastic processes on fractals*
09:40–10:20	Jun Kigami. *Time change of Brownian motions – Poincaré inequality and Protodistance*
10:20–10:50	**Coffee break**
10:50–11:30	Francesco Mainardi. *Complete monotonicity for fractional relaxation processes*
11:30–12:00	Martina Zähle. *Ordinary and partial differential equations with fractional noise*
12:00–12:30	Ciprian Tudor. *Fractional processes and the solution to the heat equation*
12:30–14:00	**Lunch**
14:00–14:40	Maria Dolorez Ruiz-Medina. *Fractional in time and in space evolution equations driven by fractional white noise*
14:40–16:00	**Discussion in groups**

Models with fractional Brownian motion	**Fractional analysis**
Wolfgang Bock	Vikash Pandey
Mikhail Zhitlukhin	Clemente Cesarano
Kamran Kalbasi	Dimiter Prodanov
Rafał Łochowski	Elena Hernández-Hernández

16:00–	**Departure by bus to boat (haven, Leiden)**
16:30–20:30	**Boat tour and workshop dinner**
21:00–	**Departure from boat to Leiden train station, Lorentz Center, or hotel Van der Valk**

Thursday, May 19

09:00–09:40	Murad Taqqu. *Behavior of the generalized Rosenblatt process at extreme critical exponent values*
09:40–10:20	Gennady Samorodnitsky. *New classes of self-similar stable processes with stationary increments and functional central limit theorems for heavy tailed stationary infinitely divisible processes generated by conservative flows*
10:20–10:50	**Coffee break**
10:50–11:30	Enzo Orsingher. *Random flights and fractional D'Alembert operators*
11:30–12:00	Martin Grothaus. *Mittag-Leffler analysis: construction and applications*
12:00–12:30	Giulia Di Nunno. *A Malliavin–Skorohod calculus in L^0 and L^1 for additive and Volterra-type processes*
12:30–14:00	**Lunch**
14:00–14:40	Nikolai Leonenko. *Limit theorems for multifractal products of geometric stationary processes*
14:40–15:20	Mark Podolskij. *Limit theorems for stationary increments Lévy moving average processes*
15:20–15:50	Tommi Sottinen. *Parameter Estimation for the Langevin Equation with Stationary-Increment Gaussian Noise*
15:50–16:10	**Coffee break**
16:10–16:40	Yuliya Mishura. *Utility maximization on the Wiener-transformable markets*
16:40–17:10	Andreas Neuenkirch. *Asymptotic stability for differential equations driven by Hölder continuous paths*
17:10–18:20	**Discussion in groups**

Interplay between stochastic and deterministic fractality	**Statistical inference of fractional models**	**Fractional analysis in applications**
Marco Dozzi	Kęstutis Kubilius	Elena Boguslavskaya
Bruno Toaldo	Vladimir Panov	Jan Korbel
Angelica Pachon Pinzon	Dmytro Marushkevych	Ivan Matychyn

Friday, May 20

09:00–09:40	Virginia Kiryakova. *Operators of generalized fractional calculus: theory, examples, applications*
09:40–10:20	Yuri Luchko. *Fractional Calculus models of the two-dimensional anomalous diffusion and their analysis*
10:20–10:50	**Coffee break**
10:50–11:30	Anatoly Kochubei. *Fractional differential systems and fractional approximation*
11:30–12:00	Grygoriy Torbin. *On fractal dimension faithfulness and its applications*
12:00–12:30	Pavel Chigansky. *The exact eigenvalues and eigenfunctions asymptotics of covariance operators related to fBm*
12:30–14:00	**Lunch**
14:00–14:30	Sverre Holm. *Connecting fractional wave equations with physics*
14:30–15:00	Georgiy Shevchenko. *Extended fractional integral and representation results for fractional Brownian motion*
15:00–16:00	**Concluding round table**

Abtracts

Antoine Ayache (Université Lille 1, France)

Stationary increments harmonizable stable fields: upper estimates on path behavior

Studying sample path behavior of stochastic fields/processes is a classical research topic in probability theory and related areas such as fractal geometry. To this end, many methods have been developed since a long time in Gaussian frames. They often rely on some underlying "nice" Hilbertian structure, and can also require finiteness of moments of high order. Therefore, they can hardly be transposed to frames of heavy-tailed stable probability distributions. However, in the case of some linear non-anticipative moving average stable fields/processes, such as the linear fractional stable sheet and the linear multifractional stable motion, rather new wavelet strategies have already proved to be successful in order to obtain sharp moduli of continuity and other results on sample path behavior. The main goal of our talk is to show that, despite the difficulties inherent in the frequency domain, such kind of a wavelet methodology can be generalized and improved, so that it also becomes fruitful in a general harmonizable stable setting with stationary increments. Let us point out that there are large differences between this harmonizable setting and the moving average stable one.

The talk is based on joint work with Geoffrey Boutard (Université Lille 1).

Ehsan Azmoodeh (University of Vaasa, Finland)

Stein's method on the second Wiener chaos

We consider general elements in the second Wiener chaos of the form

$$F_\infty = \sum_{k=1}^{q} \alpha_k (N_k^2 - 1)$$

where $\{N_k\}_{k\geq 1}$ is a sequence of i.i.d $\mathscr{N}(0,1)$ random variables, and distinct real coefficients α_k. Using Malliavin calculus techniques, we introduce a new form of the Stein's discrepancy, that is very tractable by Malliavin calculus, to provide bounds on the 2-Wasserstein distance $d_{W_2}(F_n, F_\infty)$ as soon as the sequence $\{F_n\}_{n\geq 1}$ itself comes from the second Wiener chaos. Our bound is given by finitely many cumulants, and can be seen as generalization of that of the normal approximation on Wiener space, see for example [3]. Our method of proof is entirely original. In particular it does not rely on estimation of bounds on solutions of the so-called Stein equations at the heart of Stein's method. The talk is based on the recent work [1].

[1] Arras. B., Azmoodeh. E., Poly. G., Swan. Y. (2016) *Stein's method on the second Wiener chaos: 2-Wasserstein distance*. `http://arxiv.org/abs/1601.03301`.

[2] Azmoodeh. E., Peccati, G., Poly. G. (2014) *Convergence towards linear combinations of chi-squared random variables: a Malliavin-based approach*. Séminaire de Probabilités (Special volume in memory of Marc Yor), 339-367.

[3] Nourdin. I., Peccati, G. (2012). Normal Approximations Using Malliavin Calculus: from Stein's Method to Universality. Cambridge Tracts in Mathematics. Cambridge University.

Luisa Beghin (Sapienza University of Rome, Italy)

Time-dependent fractional generators and related additive processes

Time-inhomogeneous, or additive, processes are obtained from Lévy processes by relaxing the condition of stationarity of increments. Due to the independence of their increments, additive processes are spatially (but not temporally) homogeneous Markov processes. By analogy with the case of Lévy processes, one can define an infinitesimal generator, which is, as a consequence, time-dependent. Additive versions of stable and Gamma processes have been studied, for example, in [4], [5], [6]. We consider here time-inhomogeneous generalizations of the well-known geometric stable processes, defined by means of time-dependent versions of fractional pseudo-differential operators of logarithmic type. The latter have been introduced and analyzed, in the time-homogeneous case, in [1], [2] and [3]. The corresponding local Lévy measures are expressed in terms of Mittag–Leffler functions with time-dependent parameters.

[1] Beghin L., *Geometric stable processes and fractional differential equation related to them*, Electron. Commun. Probab., 2014, 19, 13, 1-14.

[2] Beghin L., *Fractional gamma and gamma-subordinated processes*, Stoch. Anal. Appl., 2015, 33 (5), 903-926.

[3] Beghin L., *Fractional diffusion-type equations with exponential and logarithmic differential operators*, Arxiv 1601.01476v1, 2016, submitted.

[4] Cinlar E., *On a generalization of Gamma processes*, Journ. Appl. Probab., 17, 2, 1980, 467-480.

[5] Molchanov I., Ralchenko K., *Multifractional Poisson process, multistable subordinator and related limit theorems*, Stat. Probab. Lett., 96, 2015, 95-101.

[6] Orsingher E., Ricciuti C., Toaldo B., *Time-inhomogeneous jump processes and variable order operators,* Potential Analysis, 2016, to appear.

W. Bock (Technische Universität Kaiserslautern, Germany)

Recent results on fractional Edwards models

Random paths and in particular (weakly) self-avoiding paths have been studied intensively in physics as well as in probability theory. In physics they play a crucial role in the modeling of chain polymers with "excluded volume effect". In these models self-intersections are penalized; in the continuum case this is the Edwards model [2], in the discrete case one speaks of the Domb-Joyce model [1]. The penalization factor in the Edwards model is introduced through a modification of the Wiener measure μ_0 by a Gibbs factor

$$d\mu = \frac{1}{Z}\exp\left(-g\int_0^N d\tau \int_0^\tau dt\,\delta(B(\tau)-B(t))\right) d\mu_0,$$

where $N, g > 0$, δ is the d-dimensional Dirac delta function, B denotes a version of a d-dimensional Brownian motion and

$$Z = \mathbb{E}\left(\exp\left(-g\int_0^N d\tau \int_0^\tau dt\,\delta(B(\tau)-B(t))\right)\right).$$

We set

$$L \equiv \int_0^N d\tau \int_0^N dt\,\delta(B(\tau)-B(t)),$$

which is known as the "self-intersection local time" of a Brownian motion.

The one-dimensional weakly self-avoiding random walk is well understood and proofs of mathematical rigor concerning the scaling exponent can be found in [3, 4].

For fractional weakly self-avoiding paths the contrary is the case. This is due to the lack of Markov and martingale properties, which excludes many techniques of stochastic analysis.

In this talk we will summarize recent results—both numerical and analytical—on Fractional Edwards Models.

[1] C. Domb and G. S. Joyce. Cluster expansion for a polymer chain. *Journal of Physics C: Solid State Physics*, 5(9):956+, 1972.

[2] S. F. Edwards. The statistical mechanics of polymers with excluded volume. *Proc. Phys. Soc.*, 85: 613–624, 1965.

[3] A. Greven and F. den Hollander. A variational characterization of the speed of a one-dimensional self-repellent random walk. *The Annals of Applied Probability*, 3(4):1067–1099, 11 1993.

[4] R. van der Hofstad, F. den Hollander, and W. König. Central limit theorem for the Edwards model. *Ann. Probab.*, 25(2):573–597, 04 1997.

Elena Boguslavskaya (Brunel University London, UK)

Fractional Appell polynomials and their applications

Appell polynomials and their properties are well known and widely applied in probability. Here we introduce fractional Appell polynomials and discuss their properties and applications.

The motivation for the definition of fractional Appell polynomials is the following: as it was shown in [1], Appell polynomials of order n can be obtained as A-transforms of the corresponding monomials of order n. In a similar way fractional Appell polynomials of order a can be obtained as A-transforms of the corresponding monomials of order a.

Fractional Appell polynomials exhibit many nice properties similar to those of Appell polynomials. For example, fractional Appell polynomials are martingales if built on Levy processes (or some other Markovian processes). We show how fractional Appell polynomials can be applied to calculate some functionals of some Markov processes, and solve some optimal stopping problems.

[1] Boguslavskaya, E., *Solving optimal stopping problems for Levy processes in infinite horizon via A-transform*. http://arxiv.org/pdf/1403.1816.pdf

[2] Boguslavskaya, E., *Fractional Appell Polynomials*. (in preparation)

Clemente Cesarano (Università Telematica Internazionale UNINETTUNO, Italy)

Fractional operators and special polynomials

The use of the translation operators techniques combining with some classes of orthogonal polynomials, as the generalized Hermite polynomials, to describe the action of the exponential operators with fractional derivatives. Moreover, since the approach to fractional derivatives to deal with integral transform traces back to Riemann and Liouville and the combined use of ordinary transforms of Laplace or Fourier type can be exploited to define operators involving fractional power differential operators, it will study as the integral transforms providing the solution of some partial differential equations, as the heat equations, and in according with the Gauss transform, this method could be considered as a benchmark for the more general case employing some special family of generalized Hermite polynomials of the Kampé de Fériet type.

Pavel Chigansky (The Hebrew University, Israel)

Exact eigenvalues and eigenfunctions asymptotics
of the covariance operators related to fractional Brownian motion [1]

In this work we revisit the eigenvalue problem

$$(K\varphi)(t) = \lambda\varphi(t), \quad t \in (0,1)$$

for the covariance operators K, associated with the fractional Brownian motion $B^H = (B^H; t \in [0,1])$ with the Hurst parameter $H \in (0,1)$:

$$(K\varphi)(t) := \int_0^1 \frac{1}{2}\Big(t^{2H} + s^{2H} - |t-s|^{2H}\Big)\varphi(s)ds,$$

and its formal derivative:

$$(K\varphi)(t) := \frac{d}{dt}\int_0^1 f(s)|t-s|^{2H-1}\text{sign}(t-s)ds, \quad t \in (0,1).$$

Our main results are the exact asymptotic formulas for the eigenvalues and the corresponding eigenfunctions. We will discuss a number of applications, including the optimal filtering and statistical inference in linear models and computation of the exact L_2-small balls probabilities.

The talk is based on joint work with Marina Kleptsyna (Université du Maine, France).

[1] Pavel Chigansky is supported by ISF grant

[1] C. Cai, P. Chigansky, M. Kleptsyna, Mixed Gaussian processes: a filtering perspective, to appear in Ann. of Prob., preprint arXiv:1208.6253
[2] P. Chigansky, M. Kleptsyna, Spectral asymptotics of the fractional Brownian covariance operator, preprint arXiv:1507.04194
[3] P. Chigansky, M. Kleptsyna, Asymptotics of the Karhunen-Loeve expansion for the fractional Brownian motion, preprint arXiv:1601.05715

Serge Cohen (Institut de Mathématiques de Toulouse, Université Paul Sabatier, France)

Fractional fields parametrized by manifolds

Among other important applications of Self-Similarity one is quite classical for finite-dimensional Euclidean spaces : it is a convenient way to obtain generic textures used in the modeling of images. But it is obvious even for non-specialist that "textures" exist on curved spaces. The most popular model for self-similar fields is certainly the fractional Brownian field, and, maybe, even before the Lévy Brownian field. Even if the definition of a self-similar field parametrized by a Riemannian manifold is not straightforward, the definition of fractional Brownian fields parametrized by metric field is not difficult. The aim of this talk will be to review some classical facts concerning these fields and to introduce some geometrical obstructions to the existence of fractional Brownian field.

Giulia Di Nunno (University of Oslo, Norway)

A Malliavin-Skorohod calculus in L^0 and L^1 for additive and Volterra-type processes [1]

In the paper [1] we develop a Malliavin–Skorohod type calculus for additive processes in the L^0 and L^1 settings, extending the probabilistic interpretation of the Malliavin–Skorohod operators to this context. We prove calculus rules and obtain a generalization of the Clark–Haussmann–Ocone formula for random variables in L^1. Our theory is then applied to extend the stochastic integration with respect to volatility modulated Lévy-driven Volterra processes recently introduced in the literature, see e.g. [2]. Our work yields to substantially weaker conditions that permit to cover integration with respect to e.g. Volterra processes driven by α-stable processes with $\alpha < 2$. The presentation focuses on jump type processes.

This talk is based on joint work with Josep Vives, University of Barcelona.

[1] G. Di Nunno and J. Vives. A Malliavin-Skorohod calculus in L^0 and L^1 for additive and Volterra-type processes. ArXiv 1502.5631. To appear in Stochastics.
[2] O.E. Barndorff-Nielsen, F.E. Benth, J. Pedersen, and A.E.D. Veraart. On stochastic integration for volatility modulated Lévy-driven Volterra processes. Stochastic Processes and their Applications, 2014 (124), 812-847.

Mirko D'Ovidio (Sapienza University of Rome, Italy)

Delayed diffusions on random Koch domains

We consider time-changed Brownian motions on Koch (pre-fractal/fractal) domains where the time change is given by the inverse of a stable subordinator, thus the diffusion turns out to delayed. In particular, we study the fractional Cauchy problem with Robin condition on the pre-fractal boundary and obtain asymptotic results for the corresponding delayed diffusions with Robin, Neumann and Dirichlet boundary conditions on the fractal domain. Our results are obtained for random Koch domains: the mixtures of Koch curves with random scales.

This talk is based on joint work with Raffaela Capitanelli (Sapienza University of Rome).

[1] Bass R.F. and Hsu P., Some Potential Theory for Reflecting Brownian Motion in Holder and Lipschitz Domains, *The Annals of Probability*, 19:2 (1991), 486–508.

[1] G. Di Nunno thanks FINEWSTOCH ISP-project funded by the Research Council of Norway for the support.

[2] Capitanelli R. and Vivaldi M.A., Insulating Layers and Robin Problems on Koch Mixtures, *Journal of Differential Equations*, 251 (2011),1332–1353.
[3] Davies E.B., Two dimensional Riemannian manifolds with fractal boundaries, *J. London Math. Soc.* (2) 49 (1994) 343–356.
[4] Kilbas A. A., Srivastava H. M. and Trujillo J. J., *Theory and Applications of Fractional Differential Equations.* North-Holland Mathematics Studies, Vol. 204. Amsterdam: Elsevier, 2006.
[5] Banuelos R. and Pang M.M.H., Stability and approximations of eigenvalues and eigenfunctions for the Neumann Laplacian, Part 1, *Electronic Journal of Differential Equations*, (145) 2008 (2008), 1–13.
[6] Pang M.M.H., Stability and approximations of eigenvalues and eigenfunctions for the Neumann Laplacian, Part 2, *J. Math. Anal. Appl.* 345 (2008) 485–499.

Marco Dozzi (IECL, Lorraine University, France)

On the blowup behaviour of semilinear stochastic partial differential equations with fractional noise

We consider stochastic partial differential equations with multiplicative mixed (Brownian / fractional Brownian) noise $\dot{B}$ and study the existence of mild and weak solutions and their equivalence, and the large time behaviour of the solution. For semilinear boundary value problems of the type

$$\begin{aligned} du(t,x) &= (\Delta^s u(t,x) + G(u(t,x))dt + \kappa u(t,x)dB_t, \ t>0, \ x \in D \subset \mathbb{R}^d, \\ u(0,x) &= f(x) \geqq 0, \ \ u(t,x) = 0 \text{ for } (t,x) \in \mathbb{R}_+ \times \partial D, \end{aligned}$$

where $G : \mathbb{R} \to \mathbb{R}_+$ is locally Lipschitz, we give lower and upper bounds for the blowup time of u in terms of an associated random partial differential equation, obtained by the transformation $v(t,x) = u(t,x)\exp(\kappa B_t)$. Sufficient conditions for blowup in finite time and for the existence of a global solution are deduced in terms of the parameters of the equation. Extensions of these results to systems of equations of the type above will be given.

The talk is based on joint work with Rim Touibi (IECL, Lorraine University).

Uta Renata Freiberg (University of Stuttgart, Germany)

Graphs of functions with varying fractal dimension

We consider real continuous functions over the unit interval whose graphs have non constant box dimensions and hence non constant Hölder exponents. Moreover we explain how to construct such deterministic or random functions with a given dimension profile. This is a joint work with Elias Hauser from University of Stuttgart.

Martin Grothaus (University of Kaiserslautern, Germany)

Mittag–Leffler analysis: construction and applications

Motivated by the results of infinite dimensional Gaussian analysis and especially white noise analysis, we construct a Mittag–Leffler analysis. This is an infinite dimensional analysis with respect to non-Gaussian measures of Mittag–Leffler type which we call Mittag-Leffler measures. Our results indicate that the Wick ordered polynomials, which play a key role in Gaussian analysis, cannot be generalized to this non-Gaussian case. We provide evidence that a system of biorthogonal polynomials, called generalized Appell system, is applicable to the Mittag–Leffler measures, instead of using Wick ordered polynomials. With the help of an Appell system, we introduce a test function and a distribution space. Furthermore we give characterizations of the distribution space and we characterize the weak integrable functions and the convergent sequences within the distribution space. We construct Donsker's delta in a non-Gaussian setting as an application. In the second part, we develop a grey noise analysis. This is a special application of the Mittag–Leffler analysis. In this framework, we introduce generalized grey Brownian motion and prove differentiability in a distributional sense and the existence of generalized grey Brownian motion local times. Grey noise analysis is then applied to the time-fractional heat equation and the time-fractional Schrödinger equation. We prove a generalization of the fractional Feynman–Kac formula for distributional initial values. In this way, we find a Green's function for the time-fractional heat equation which coincides with the solutions given in the literature. The results of this talk are published in [1], [2]

[1] M. Grothaus; F. Jahnert; F. Riemann; J. L. da Silva. Mittag–Leffler analysis I: Construction and characterization. Journal of Functional Analysis. 268(7), 1876–1903, 2015.

[2] M. Grothaus; F. Jahnert. Mittag–Leffler analysis II: Application to the fractional heat equation. Journal of Functional Analysis, 270(7), 2732-–2768, 2016.

M. Elena Hernández-Hernández (University of Warwick, UK)

Well-posedness results for two-sided generalized fractional equations [1]

We study the well-posedness and regularity of two-sided generalized fractional equations involving Riemann-Liouville type and Caputo type operators, which include equations of the form

$$\omega_1(t)D^{\beta_1(t)}_{a+*}u(t) + \omega_2(t)D^{\beta_2(t)}_{b-*}u(t) = \lambda u(t) + \gamma(t)u'(t) + \alpha(t)u''(t), \quad t \in (a,b)$$
$$u(a) = u_a, \quad u(b) = u_b,$$

where $D^{\beta_1(t)}_{a+*}$ (resp. $D^{\beta_2(t)}_{b-*}$) is the Caputo (resp. Riemann-Liouville) derivative of variable order $\beta(t) \in (0,1)$. We use a probabilistic approach which supplies stochastic representations for the corresponding solutions. These results extend those presented by the authors in [1], [2].

The talk is based on joint work with Vassili Kolokoltsov (University of Warwick).

[1] Hernández-Hernández M. E., Kolokoltsov, V. N., *On the probabilistic approach to the solution of generalized fractional differential equations of Caputo and Riemann-Liouville type*. Journal of Fractional Calculus and Applications, Vol. 7 (1), Jan. 2016, pp. 147-175.

[2] Hernández-Hernández M. E., Kolokoltsov, V. N., *Probabilistic solutions to nonlinear fractional differential equations of generalized Caputo and Riemann-Liouville type* (submitted).

Michael Hinz (University of Bielefeld, Germany)

Vector analysis on fractals and notions of dimension

We consider items of vector analysis on metric measure spaces, with a particular interest in fractal spaces. In contrast to heat conduction, which is governed by the (real valued) Hausdorff and spectral dimension, vector equations are dominated by the (integer valued) topological and martingale dimension. On Riemannian manifolds all these dimensions agree, but on fractals they typically disagree. We discuss some consequences of discrepancies of dimensions from the point of view of vector analysis.

The talk is based on joint work with Alexander Teplyaev (University of Connecticut, USA).

Sverre Holm (University of Oslo, Norway)

Connecting fractional wave equations with physics

Fractional wave equations are of interest for modeling wave propagation in e.g. medical ultrasound and medical elastography. In an attempt to justify a closer link to the physics we have highlighted four ways where this connection is apparent [1]. These are 1) the ability to model power

[1] M. Elena Hernández-Hernández is supported by Chancellor's International Scholarship and Department of Statistics through the University of Warwick, UK

law attenuation, 2) their root in constitutive equations, 3) the link with an weighted sum of elementary relaxation processes, and 4) the possibility to express fractional derivatives with higher order derivatives. Of these 2) and 3) will be discussed here. The Kelvin–Voigt wave equation for the compressional wave may be derived from a grain-to-grain shearing process from sediment acoustics [2]. Likewise the fractional diffusion-wave equation describes the shear wave in this model [3]. This is therefore one of the few derivations of fractional constitutive equations from first principles. The relaxation sum formulation is also of interest because the weights in this sum turn out to have a long-tailed, i.e. fractal distribution [4]. The distribution has in general two power law asymptotes and thus describes a multi-fractal. This hints at an indirect underlying fractality in the medium which causes behavior described by the fractional derivatives [1].

[1] S. Holm, Four ways to justify temporal memory operators in the lossy wave equation, in *Proc. IEEE Ultrasonics Symposium*. Taipei, Taiwan, 2015.

[2] M. J. Buckingham, Wave propagation, stress relaxation, and grain-to-grain shearing in saturated, unconsolidated marine sediments, *J. Acoust. Soc. Am.*, vol. 108, no. 6, pp. 2796–2815, 2000.

[3] V. Pandey and S. Holm, Connecting the grain-shearing mechanism of wave propagation in marine sediments to fractional calculus, *submitted for publication*, 2015.

[4] S. P. Näsholm and S. Holm, "Linking multiple relaxation, power-law attenuation, and fractional wave equations," *J. Acoust. Soc. Am.*, vol. 130, no. 5, pp. 3038–3045, 2011.

Kamran Kalbasi (University of Warwick, UK)

Anderson polymer in fractional Brownian environment [1]

We consider the Anderson polymer partition function

$$u(t) := \mathbb{E}^X \left[e^{\int_0^t B(\mathrm{d}s, X_s)} \right],$$

where $\{B(t,x); t \geq 0\}_{x \in \mathbb{Z}^d}$ is a family of independent (time) fractional Brownian motions indexed by $\mathbb{Z}^d$, $\{X_t\}_{t \in \mathbb{R}^+}$ is a continuous-time simple symmetric random walk on $\mathbb{Z}^d$ and started from the origin, and $\mathbb{E}^X$ denotes expectation with respect to this random walk. In this talk, I will present our recent results on the quenched and annealed asymptotic behavior of $\frac{1}{t} \log u(t)$.

[1] K. Kalbasi, T. S. Mountford, and F. G. Viens Frederi, *Asymptotic behavior of the Anderson polymer in a fractional Brownian environment*. arXiv preprint arXiv:1602.05491, 2016.

Jun Kigami (Kyoto University, Japan)

Time change of Brownian motions – Poincaré inequality and Protodistance

In this talk, time change of the Brownian motion of the 2-dim. square is considered. Time change corresponds to the introduction of inhomogeneity of medium. As a consequence, after time change the process has the same paths as the Brownian motion but it can have different speed at each point. Recently, time change with respect to random measures such as Liouville measure, or Liouville quantum gravity has been of much interest in relation with associated random geometry. We will define a class of measures called measures with weak exponential decay, which includes Liouville measure, and show the existence of associated time changed process which posesses a jointly continuous heat kernel. To give an estimate of heat kernel, we will introduce the notion of "protodistance", which is a candidate of proper substitute of intrinsic metric.

[1] Kamran Kalbasi is supported by Swiss National Science Foundation

Virginia Kiryakova (Institute of Mathematics and Informatics, Bulgarian Academy of Sciences)

Operators of generalized fractional calculus: theory, examples, applications

We survey the theory of the Generalized Fractional Calculus (GFC) [1], the operators of generalized differentiation and integration of fractional (arbitrary) multi-order $(\delta_1,\ldots,\delta_m)$. Based on commuting compositions of Erdélyi-Kober fractional integrals

$$I_{\beta_k}^{\gamma_k,\delta_k} f(x) = \int_0^1 \frac{(1-\sigma)^{\delta_k-1}}{\Gamma(\delta_k)} \sigma^{\gamma_k} f\left(x\sigma^{1/\beta_k}\right) d\sigma, \text{ of order } \delta_k > 0, \text{ with } \gamma_k \in \mathbb{R}, \beta_k > 0,$$

$k = 1,2,\ldots,m$, we introduced the *generalized fractional integrals* $I_{(\beta_k),m}^{(\gamma_k),(\delta_k)} = \prod_{k=1}^m I_{\beta_k}^{\gamma_k,\delta_k}$, but defined by means of single integral operators with special functions as kernels: Fox's H-function (or Meijer's G-function if $\forall \beta_k = \beta$). For $\sum_{k=1}^m \delta_k > 0$, these are as:

$$I_{(\beta_k),m}^{(\gamma_k),(\delta_k)} f(x) := \int_0^1 H_{m,m}^{m,0} \left[\sigma \left| \begin{array}{c} (\gamma_k + \delta_k + 1 - 1/\beta_k, 1/\beta_k)_1^m \\ (\gamma_k + 1 - 1/\beta_k, 1/\beta_k)_1^m \end{array} \right. \right] f(x\sigma) d\sigma,$$

and if $\forall \delta_k = 0$, these are identity operators. The corresponding *generalized fractional derivatives* $D_{(\beta_k),m}^{(\gamma_k),(\delta_k)}$ are defined by means of explicit differ-integral expressions, using the idea for the Riemann-Liouville and Erdélyi-Kober fractional derivatives D^δ, $D_\beta^{\gamma,\delta}$ but involving polynomials of $x\frac{x}{dx}$, see [1, 2], etc. To this end, and in developing all operational properties of GFC operators $I_{(\beta_k),m}^{(\gamma_k),(\delta_k)}$, $D_{(\beta_k),m}^{(\gamma_k),(\delta_k)}$, the theory of H- and G-functions is essentially used. *Caputo-type* generalized fractional derivatives ${}^C D_{(\beta_k),m}^{(\gamma_k),(\delta_k)}$ are also introduced ([3]), and the relation of GFC operators with the Gel'fond-Leontiev generalized integrations and differentiations is shown. Several *examples*, known and new *applications* of the GFC are discussed. In particular, the hyper-Bessel differential operators of arbitrary integer order $m > 1$ are shown as special cases of multiorder $(1,1,\ldots,1)$, as well as the classical FC operators and many other Calculus' operators. Some of the results come from parallel or joint works with Yu. Luchko.

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[2] Yakubovich, S. and Luchko, Yu. The Hypergeometric Approach to Integral Transforms and Convolutions. Kluwer, 1994.

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Anatoly N. Kochubei (Institute of Mathematics, National Academy of Sciences of Ukraine)

Fractional differential systems and fractional approximation

We develop a theory of the Cauchy problem for two classes of linear evolution systems of partial differential equations with the Caputo-Djrbashian fractional derivative of order $\alpha \in (0,1)$ in the time variable t. The class of fractional-parabolic systems is a fractional extension of the class of systems of the first order in t satisfying the uniform strong parabolicity condition. We construct and investigate the Green matrix of the Cauchy problem, prove a uniqueness result.

For the class of fractional-hyperbolic systems, which can be seen as a fractional analogue of the class of hyperbolic systems, we construct a fundamental solution of the Cauchy problem having exponential decay outside the fractional light cone $\{(t,x) : |t^{-\alpha}x| \le 1\}$.

We consider also an application of fractional equations to the study of singular first order evolution equations and systems. Their study in scales of Banach spaces, via the so-called Ovsyannikov method, often gives a solution only on a finite time interval. We show how to approximate a solution of such an equation, together with its possible analytic continuation, using a solution (existing for all $t > 0$) of the time-fractional equation of order $\delta > 1$, where $\delta \to 1 + 0$.

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[2] Kochubei, A. N., *Fractional-hyperbolic systems*, Fract. Calc. Appl. Anal. 16, No. 4 (2013), 860–873.
[3] Kochubei, A. N. and Kondratiev, Yu. G., *Fractional approximation of solutions of evolution equations*, Analysis (to appear), DOI: 10.1515/anly-2015-5007, ArXiv 1504.04840.

Vassili Kolokoltsov (University of Warwick, UK)

Probablistic approach for solving fractional differential equations of Caputo type and their various extensions.

The probabilistic treatment of the Caputo derivative (suggested recently by the author) allows one to carry out an effective, systematic and concise analysis of various kinds of well known problems and their far reaching generalizations. This claim will be demonstrated on various classes of equations.

Yuri Kondratiev (University of Bielefeld, Germany)

Fractional stochastic dynamics in the continuum

We start with Markov dynamics of interacting particle systems in the continuum. A fractional time evolution in such systems corresponds to random time changes. In the Vlasov type scaling, it leads to a fractional mesoscopic hierarchy for correlation functions. Corresponding state evolutions are obtained by means of a subordination of Poisson flows (which describe the kinetic behavior of initial Markov dynamics). We will discuss subordination effects, in particular, the notion of intermittency which appear as a result of fractional evolution and never is possible in the Markov kinetics.

Jan Korbel (Czech Technical University in Prague and Max-Planck-Institute for the History of Science, Berlin, Germany)

Option pricing beyond Black–Scholes based on double–fractional diffusion

e present a novel option pricing model based on spatio-temporal double-fractional diffusion process. We start with several generalizations of differential calculus for derivatives of non-natural order, including Caputo derivative and Riesz-Feller derivative. We discuss properties of processes driven by double-fractional diffusion equations and their possible representations. Subsequently, we apply the model to option pricing and obtain a formula depending only on a three parameters. We analyze the model on real data of S&P 500 options traded in November 2008 and compare it with Black-Scholes model and Lévy stable option pricing model. Salient issues as model calibration scheme or applications to hedging are also discussed. The work was recently published in Ref. [1].

The talk is based on joint work with Hagen Kleinert (Free University of Berlin and ICRANet, Italy).

[1] H. Kleinert and J. Korbel. Physica A **449** (2016), 200-214.

Kęstutis Kubilius (Vilnius University, Lithuania)

On statistical inference for SDE driven by fractional Brownian motion

Assume that observable continuous time process $(X_t)_{t\in[0,T]}$ satisfies SDE

$$X_t = \xi + \int_0^t f(X_s)\,ds + \int_0^t g(X_s)\,dB_s^H, \quad t \in [0,T],$$

where $T > 0$ is fixed, ξ is an initial r.v., f and g are continuous functions satisfying some regularity conditions, and B^H is a fBm with the Hurst index $1/2 < H < 1$.

Our goal is to construct a strongly consistent and asymptotically normal estimators of the H and the local variance function g from discrete observations $X_{t_1},\dots,X_{t_n}$ of trajectory X_t, $t \in [0,T]$.

These estimators are based on the second order increments of an observed discrete trajectory. We also compare the asymptotic behavior of these estimators with the aid of computer simulations.

Moreover, some new estimators are suggested and application of our methods to the stochastic differential equations driven by Gaussian process are discussed.

[1] Kubilius K., Skorniakov V., Melichov D., Estimation of parameters of SDE driven by fractional Brownian motion with polynomial drift, Journal of Statistical Computation and Simulation, http://dx.doi.org/10.1080/00949655.2015.1095301 (online).

[2] Kubilius K., Skorniakov V., On some estimators of the Hurst index of the solution of SDE driven by a fractional Brownian motion. // Statistics and Probability Letters, 109, 2016, p. 159-167.

[3] Kubilius K., Skorniakov V., A short note on a class of statistics for estimation of the Hurst index of fractional Brownian motion. Preprint 2016.

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Takashi Kumagai (Kyoto University, Japan)

Time changes of stochastic processes on fractals [1]

In recent years, interest in time changes of stochastic processes according to irregular measures has arisen from various sources. One typical example of such time-changed processes include the so-called *Fontes-Isopi-Newman (FIN) diffusion* [2], the introduction of which was motivated by the study of the localization and aging properties of physical spin systems. This FIN diffusion is known to be the scaling limit of the one-dimensional Bouchaud trap model [1, 2].

We will provide a general framework for studying such time changed processes and their discrete approximations in the case when the underlying stochastic process is strongly recurrent, in the sense that it can be described by a resistance form, as introduced by J. Kigami. In particular, this includes the case of Brownian motion on tree-like spaces and low-dimensional self-similar fractals. If time permits, we also discuss heat kernel estimates for the relevant time-changed processes.

The talk is based on joint work with David Croydon (University of Warwick, UK) and Ben Hambly (Mathematical Institute, Oxford, UK).

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Joachim Lebovits (Université Paris 13 Nord, France)

Stochastic calculus with respect to Gaussian processes

Stochastic integration with respect to Gaussian processes, such as fractional Brownian motion (fBm) or multifractional Brownian motion (mBm), has raised strong interest in recent years, motivated in particular by applications in finance, Internet traffic modeling and biomedicine. The aim of this work to define and develop, using White Noise Theory, an anticipative stochastic calculus with respect to a large class of Gaussian processes, denoted $\mathscr{G}$, that contains, among many other processes, Volterra processes (and thus fBm) and also mBm. This stochastic calculus includes a definition of a stochastic integral, Itô formulas (both for tempered distributions and for functions with sub-exponential growth), a Tanaka Formula as well as a definition, and a short study, of (both weighted and non weighted) local times of elements of $\mathscr{G}$.

In that view, a white noise derivative of any Gaussian process G of $\mathscr{G}$ is defined and used to integrate, with respect to G, a large class of stochastic processes, using Wick products. A comparison of our integral *wrt* elements of $\mathscr{G}$ to the ones provided by Malliavin calculus in [1] and by Itô stochastic

[1]The research of Takashi Kumagai is supported in part by JSPS KAKENHI Grant Number 25247007.

calculus is also made. Moreover, one shows that the stochastic calculus with respect to Gaussian processes provided in this work generalizes the stochastic calculus originally proposed for fBm in [2, 3, 4] and for mBm in [5, 6, 7]. Likewise, it generalizes results given in [8] and some results given in [1]. In addition, it offers alternative conditions to the ones required in [1] when one deals with stochastic calculus with respect to Gaussian processes.

A particular focus will be given to stochastic calculus with respect to fractional and multifractional Brownian motions.

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Nikolai N. Leonenko (Cardiff University, UK)

Limit theorems for multifractal products of geometric stationary processes [1]

Multifractal and monofractal models have been used in many applications in hydrodynamic turbulence, finance, computer network traffic, etc. (see, for example, [7]). There are many ways to construct random multifractal models ranging from simple binomial cascades to measures generated by branching processes and the compound Poisson process ([4, 7, 9]).

We investigate the properties of multifractal products of geometric Gaussian processes with possible long-range dependence and geometric Ornstein–Uhlenbeck processes driven by Lévy motion and their finite and infinite superpositions. We present the general conditions for the $\mathscr{L}_q$ convergence of cumulative processes to the limiting processes and investigate their q-th order moments and Rényi functions, which are nonlinear, hence displaying the multifractality of the processes as constructed. We also establish the corresponding scenarios for the limiting processes, such as log-normal, log-gamma, log-tempered stable or log-normal tempered stable scenarios.

This is joint work with Denis Denisov (Manchester University, UK).

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[1] N. Leonenko was supported in particular by Cardiff Incoming Visiting Fellowship Scheme and International Collaboration Seedcorn Fund, Australian Research Council's Discovery Projects funding scheme (project number DP160101366) and grant of Ministry of Education and Science of Spain, MTM2015-71839-P.

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Jacques Lévy Véhel (Anja Team, Inria, Nantes, France)

Processes with varying local regularities

This talk will concentrate on two measures of local regularity, namely the pointwise Hölder exponent and the local intensity of jumps.

I will introduce various kinds of self-regulating and self-stabilizing processes: a stochastic process X is called self-regulating if there exists a smooth deterministic function g such that, almost surely, at each time t, $h_X(t) = g(X(t))$ where $h_X(t)$ is the pointwise Hölder exponent of X at time t. Likewise, a process is termed self-stabilizing if, almost surely, at each time t, $\alpha_X(t) = g(X(t))$ where $\alpha_X(t)$ is the local intensity of jumps of X at t.

I will also present some statistical results on the estimation of the self-regulating function g, as well as applications in selected fields: finance, biomedicine and geophysics.

Rafał M. Łochowski (Warsaw School of Economics, Poland)

Integrals driven by irregular signals in Banach spaces and rate-independent characteristics of their irregularity

Integrals driven by irregular signals, like paths of fractional Brownian motions (fBms), are usually characterized by the same degrees of irregularity as these signals. There exist many different definitions of the integrals driven by fBms and for a good review of different approaches see for example [2]. Interesting, rate-dependent approach, based on fractional calculus is presented in [3]. The best rate-independent characterization of the irregularity of paths of a fBm B^H with the Hurst parameter $H \in (0;1)$, given by $\psi-$variation, is also known and in [2] it is proven that the same rate characterizes integrals driven by B^H for $H \in (\frac{1}{2};1)$. During the talk I will present another rate-independent approach, based on the functional called *truncated variation*, see [1]. The results which may be obtained from this approach for fBms are less precise than those stated in [2], but they may be applied to more general signals, also attaining their values in Banach spaces.

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Yuri Luchko (Beuth Technical University of Applied Sciences Berlin, Germany)

Fractional Calculus models of the two-dimensional anomalous diffusion and their analysis

In this talk, the basic continuous time random walk micro-model is first employed to derive a macro-model for the two-dimensional anomalous diffusion in terms of a time- and space-fractional partial differential equation with the Caputo time-fractional derivative of order $\alpha/2$ and the Riesz space-fractional derivative of order α. For $\alpha < 2$, this α-fractional diffusion equation describes the so called Lévy flights that correspond to the continuous time random walk model, where both the mean waiting time and the jump length variance of the diffusing particles are divergent. It turns out that the fundamental solution to the α-fractional diffusion equation can be interpreted as a two-dimensional probability density function. Moreover, the fundamental solution can be expressed in explicit form in terms of the Mittag–Leffler function that depends on the auxiliary variable $|x|/(2\sqrt{t})$ just as in the case of the fundamental solution to the classical isotropic diffusion equation. Remarkably, the entropy production rate associated with the anomalous diffusion process described by the α-fractional diffusion equation appears to be exactly the same as in the case of the classical isotropic diffusion equation. Thus the α-fractional diffusion equation can be considered to be a natural generalization of the classical isotropic diffusion equation that exhibits some characteristics of both anomalous and classical diffusion.

[1] Yu. Luchko, *Entropy Production Rate of a One-Dimensional Alpha-Fractional Diffusion Process*. Axioms 5, 1, 2016, doi:10.3390/axioms5010006.

[2] Yu. Luchko, *A new fractional calculus model for the two- dimensional anomalous diffusion and its analysis*. Accepted for publication in Mathematical Modelling of Natural Phenomena.

Francesco Mainardi (University of Bologna (Alma Mater), Italy)

Complete monotonicity for fractional relaxation processes [1]

We revisit some models of anomalous relaxation based on evolutions equations of fractional order. Our attention is on the complete monotonicity of the functions characterizing the relaxation processes, both in viscoelastic and dielectric media. The monotonicity requirement is known to provide a sufficient condition for the total dissipation of energy.

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Dmytro O. Marushkevych (Université du Maine, France)

Large deviations for drift parameter estimator of mixed fractional Ornstein-Uhlenbeck process

We consider the Ornstein–Uhlenbeck process driven by mixed fractional Brownian motion:

$$dX_t = -\vartheta X_t dt + d\widetilde{B}_t, \quad t \in [0,T], \quad T > 0, \quad \vartheta > 0.$$

Maximum likelihood drift parameter estimator $\widehat{\vartheta}_T(X)$ is proved to be asymptotically normal [1]. We developed this result by proving the large deviation principle for $\widehat{\vartheta}_T(X)$.

[1] This research is carried out in the framework of the activities of the National Group of Mathematical Physics (GNFM, INdAM) and of the Interdepartmental Center "L. Galvani" for integrated studies of Bioinformatics, Biophysics and Biocomplexity of the University of Bologna.

Theorem 1. *The maximum likelihood drift parameter estimator of mixed fractional Ornstein–Uhlenbeck process satisfies the large deviation principle with the good rate function*

$$I(x) = \begin{cases} -\dfrac{(x+\vartheta)^2}{4x}, & \textit{if } x < -\dfrac{\vartheta}{3}, \\ 2x+\vartheta, & \textit{otherwise}. \end{cases}$$

[1] P. Chigansky and M. Kleptsyna. *Spectral asymptotics of the fractional Brownian motion covariance operator*. arXiv preprint 1507.04194, 2015.

[2] C. Cai, P. Chigansky and M. Kleptsyna. *Mixed Gaussian processes: a filtering approach*. To appear in Ann. Probab., arXiv preprint 1208.6253, 2015.

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Ivan I. Matychyn (Kazimierz Wielki University in Bydgoszcz, Poland)

Time-optimal Control of Linear Equations with Riemann–Liouville Fractional Derivatives

Problem of time-optimal control of linear systems with fractional dynamics is treated in the paper from the convex-analytic standpoint. The following linear system of fractional differential equations involving Riemann–Liouville derivatives [1] is considered

$$D^{\alpha} z = Az + u, \quad 0 < \alpha \le 1,\ z, u \in \mathbb{R}^n,$$

under the initial conditions

$$J^{1-\alpha} z\big|_{t=0} = z^0.$$

A method to construct a control function u that brings trajectory of the system to a strictly convex terminal set in the shortest time is proposed in terms of attainability sets and their support functions. The proposed method [2] uses technique of set-valued maps and represents a fractional version of Pontryagin's maximum principle.

Theoretical results are supported by examples. For an equation of order 2α, $0 < \alpha \le 1$, it is shown that the proposed method results in a "bang-bang" optimal control, which is similar to the integer-order case.

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Yuliya Mishura (Taras Shevchenko National University of Kyiv, Ukraine)

Utility maximization in Wiener-transformable markets

We consider a utility maximization problem in a broad class of markets. Apart from traditional semimartingale markets, our class of markets includes processes with long memory, fractional Brownian motion and related processes, and, in general, Gaussian processes satisfying certain regularity conditions on their covariance functions. Our choice of markets is motivated by the well-known phenomena of the so-called "constant" and "variable depth" memory observed in real world price processes, for which fractional and multifractional models are the most adequate descriptions. We introduce the notion of a Wiener-transformable Gaussian process, and give examples of such processes, and their representations. The representation for the solution of the utility maximization problem in our specific setting is presented for various utility functions.

This is a joint work with Elena Boguslavskaya (Brunel University London, UK).

Andreas Neuenkirch (Universität Mannheim, Mannheim, Germany)

Asymptotical stability of differential equations driven by Hölder–continuous paths

In this talk, we establish asymptotic local exponential stability of the trivial solution of differential equations driven by Hölder–continuous paths with Hölder exponent greater than 1/2. This applies in particular to stochastic differential equations driven by fractional Brownian motion with Hurst parameter greater than 1/2. We also give an example of a scalar equation, where global stability of the trivial solution can be obtained.

This is joint work with M.J. Garrido-Atienza (Sevilla) and B. Schmalfuß (Jena).

Enzo Orsingher (Sapienza Università di Roma, Italy)

Random flights governed by fractional D'Alembert operators

We consider the fractional equations of the form

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \frac{\partial^2}{\partial x^2}\right)^{\alpha} u(x,t) = \lambda u(x,t), \quad 0 < \alpha < 1, \tag{1}$$

obtained by taking the fractional power of the D'Alembert operator in the Klein-Gordon equation. For $\alpha = 1$ this equation is related to the classical telegraph process. The analysis of (1) can be done by applying the McBride theory on fractional powers of hyper-Bessel operators of which some hints are provided. The representation of fractional powers in terms of Erdélyi–Kober integrals (generalizing the classical Riemann–Liouville integrals) is the main tool of our analysis. We are able to construct fractional telegraph processes $T^{\alpha}(t)$, $0 < \alpha < 1$, whose distributions are related to (1). The distribution obtained coincides for $\alpha = 1$ to the well-known law of the telegraph process and its conditional distributions coincide to those derived by using the order statistics approach. The same tools permit us to obtain the fractional planar random flight, the conditional and unconditional laws and the inhomogeneous equation governing the distribution. Hints are given for fractional random flights in Euclidean spaces of dimension $d > 2$.

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[2] Garra, R., Orsingher, E. and Polito, F., *Fractional Klein-Gordon equations and related stochastic processes*. Journal of Statistical Physics, 155, 4, 2014, 777–809.

[3] McBride, A., *Fractional Powers of a Class of Ordinary Differential Operators*, Proceedings of the London Mathematical Society, 3, 45, 1982, 519–546.

Angelica Pachon Pinzon (University of Turin, Italy)

Random graphs and their connections to continuous time stochastic processes [1]

We prove a weak convergence of the Barabási–Albert random graph to a modified Yule model, where the Yule process that represents the appearance of in-links must be conditioned to start with a fix number of individuals. Through this relation we explain why asymptotic properties of a random vertex in the Barabási—Albert model, coincide with the asymptotic properties of a random genus in the Yule model. As a by-product of our analysis, we obtain the explicit expression of the limit degree distribution for the Barabási-Albert model. This result is given in [2] using combinatoric techniques.

References to traditional and recent result of Preferential attachment models are also discussed.

This is a joint work with Federico Polito and Laura Sacerdote (University of Turin).

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Vikash Pandey (University of Oslo, Norway)

Connecting fluid-flow in fractal medium to fractional calculus

Our study builds on the work of Butera and Paola [Ann. Phys. (2014)] and introduces some amendments to their treatment. We also give physical arguments to justify our amendments. Three-dimensional fractal structures; Menger sponge and Sierpinski gasket are considered for investigation, and it is found that velocity and flux of the fluid flowing in the medium follows power-law in time. Numerical simulations of the result matches with the predictions from the Bernoulli equation for the medium. Further, the power-law dependency of the fluid-flow paves the way for connecting the process to fractional calculus which has already been extensively studied in our previous works (Pandey and Holm, 2015). Fractional-order differential equations are obtained which govern the dynamics of fluid-flow in the fractal medium. Moreover, a physical interpretation of the fractional-order is inferred in this work.

The overall goal is intended to show that the mathematical framework of fractional calculus can be used to model fluid-flow in materials such as rocks, groundwater networks and biological membranes which exhibit fractality of varying order dependent on their complexity.

This talk is based on joint work with Sven Peter Näsholm and Sverre Holm (University of Oslo).

[1] S. Butera, M. Di Paola, "A physically based connection between fractional calculus and fractal geometry," Ann. Phys. 350, 146-158 (2014).

[2] V. Pandey, S. Holm, "Connecting the grain-shearing mechanism of wave propagation in marine sediments to fractional calculus," submitted to the J. Acoust. Soc. Am. (2015).

Vladimir Panov (Higher School of Economics, Russia)

Statistical inference for fractional Lévy processes and related models

This talk is devoted to statistical inference for the integrals in the following form: $Z_t = \int_{\mathbb{R}} K(t,s)dL_s$, where $K(t,s): \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ is a deterministic function, and L_t is a Lévy process. Popular partial cases of this model are the fractional Lévy process (see [1]) and the well-balanced Ornstein-Uhlenbeck process (see [2]).

In this research we study the problem of statistical inference for the Lévy density $\nu(\cdot)$ of the process L_t from the observations of the process Z_t. The proposed estimator is based on the Mellin transform techniques. The key point in this approach is that the Mellin tranform of the derivative of the function $\Lambda(u) = \log(\mathbb{E}[e^{\imath u Z_t}])$ and the Mellin transform of the function $\bar{\nu}(x) := x\nu(x)$ are related by the following formula:

$$M[\Lambda'](z) = \mathrm{i}\Gamma(z)e^{i\pi z/2} \cdot M[\overline{\nu}](1-z).$$

This formula yields that the Mellin transform of $\overline{\nu}$, and therefore ν itself, can be estimated by getting use of the estimator of $M[\Lambda']$, which is based in its turn by the natural non-parametric estimator of the Fourier transform of Z_t. As the result of this study, we construct a consistent estimator for the Lévy density $\nu(\cdot)$, derive the convergence rates and prove their optimality. We derive also the mixing properties of the process Z_t, which draw particular interest from the theoretical point of view.

This talk is based on joint work with Denis Belomestny (Duisburg-Essen University, Germany).

[1] Marquardt, T.*Fractional Lévy processes with an application to long memory moving average processes*. Bernoulli, 12(6), 2006, 1099-1126.

[2] Schnurr, A., and Wörner, J. *Well-balanced Lévy driven Ornstein–Uhlenbeck processes.* Stat. Risk Model. , 28(4), 2011, 343-357.

Mark Podolskij (Aarhus University, Denmark)

Limit theorems for stationary increments Lévy driven moving averages

In this paper we present some new limit theorems for power variation of kth order increments of stationary increments Lévy driven moving averages. In this infill sampling setting, the asymptotic theory gives very surprising results, which (partially) have no counterpart in the theory of discrete moving averages. More specifically, we will show that the first order limit theorems and the mode of convergence strongly depend on the interplay between the given order of the increments, the considered power $p > 0$, the Blumenthal–Getoor index $\beta \in (0,2)$ of the driving pure jump Lévy process L and the behaviour of the kernel function g at 0 determined by the power α. First order asymptotic theory essentially comprise three cases: stable convergence towards a certain infinitely divisible distribution, an ergodic type limit theorem and convergence in probability towards an integrated random process. We also prove the second order limit theorem connected to the ergodic type result. When the driving Lévy process L is a symmetric β-stable process we obtain two different limits: a central limit theorem and convergence in distribution towards a $(1-\alpha)\beta$-stable random variable.

This talk is based on joint work with Andreas Basse-O'Connor (Aarhus University) and Raphaël Lachièze-Rey (Université Paris Descartes, France).

Dimiter Prodanov (Imec, Leuven, Belgium)

Regularization of derivatives on non-differentiable points [1]

A central notion of physics is the rate of change, which is idealized by the concept of derivative. On the other hand, the *derivative* captures only the locally-linear growth, while strongly non-linear change leads to divergence of the derivative. Classical physical variables, such as velocity or acceleration, are considered to be differentiable functions of position. The relaxation of the differentiability assumption could open new avenues in describing physical phenomena, for example, using the **scale relativity theory** developed by Nottale [1], which assumes fractality of space-time. Cherbit [3] introduced the notion of *α-fractional (fractal) velocity* as the limit of the fractional difference quotient. His main goal was the study of fractal phenomena and physical processes for which the instantaneous velocity was not well defined. The fractional difference can be generalized to mixed orders [5]:

Definition 2 (Mixed-order velocity)**.** Define the *fractional velocity* of mixed order $n+\beta$ of function $f(x) \in \mathbb{C}^n$ as

$$v_{\pm}^{n+\beta} f(x) := (\pm 1)^{n+1}(n+1)! \lim_{\epsilon \to 0} \frac{f(x \pm \epsilon) - T_n(x, \pm\epsilon)}{\epsilon^{n+\beta}}$$

where $n \in \mathbb{N}$; $\epsilon > 0, 0 < \beta \leq 1$ are real parameters and $T_n(x,\epsilon)$ is the Taylor polynomial $T_n(x,\epsilon) = f(x) + \sum_{k=1}^{n} \frac{f^{(k)}(x)}{k!}\epsilon^k$.

Some of the most peculiar properties of α-velocity are that it takes discrete values, it is discontinuous and non-vanishing only at points where the usual derivative is unbounded[2, 4]. In some applications it transpires that **fractional velocity** acts as an auxiliary object with regard to integer-order derivatives. Paradoxically, the irregularity of the fractional velocity can be used to regularize the usual derivatives at singular points. This can be demonstrated in the regularization procedure for the derivatives of Hölder functions, which allows for removal of the weak singularity in the derivative caused by strong non-linearities:

Definition 3. Regularized derivative of a function is defined as:

$$\frac{\bar{d}^{\,\beta\pm}}{dx} f(x) := \lim_{\epsilon \to 0} \frac{\Delta_{\epsilon}^{\pm}[f](x) - v_{+}^{\beta} f(x) \epsilon^{\beta}}{\epsilon}$$

We will require as usual that the forward and backward regularized derivatives be equal for a uniformly continuous function.

[1] The work of Dimiter Prodanov has been supported in part by a grant from Research Fund - Flanders (FWO), contract numbers 0880.212.840, VS.097.16N.

Then the following statement holds: Let $f(x,w) \in \mathbb{C}^2$ be composition with $w(x)$, a Hölder function $\mathbb{H}^{1/q}$ at x, then

$$\frac{đ^{\pm}}{dx} f(x,w) = \frac{\partial f}{\partial x} + \frac{đ^{\pm}}{dx} w(x) \cdot \frac{\partial f}{\partial w} + \frac{(\pm 1)^q}{q!} [w^q]^{\pm} \cdot \frac{\partial^q f}{\partial w^q}$$

where $[w^q]^{\pm} = \lim_{\epsilon \to 0} \left(\upsilon^{\pm\epsilon}_{1/q} [w](x) \right)^q$ is the fractal q-adic (co-)variation. Possible applications of presented approach are regularizations of quantum mechanical paths and Brownian motion trajectories, which are Holder 1/2.

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[2] Adda F B and Cresson J 2001 *J. Math. Anal. Appl.* **263** 721 – 737

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[4] Prodanov D 2015 *Fract. Calc. Appl. Anal.* **18** 580 – 602

[5] Prodanov D 2016 *ArXiv*, 1508.06086

M. Dolorez Ruiz-Medina (Granada University, Spain)

Fractional in time and in space evolution equations driven by fractional integrated white noise [1]

We study the weak-sense solution to the following fractional in time and in space stochastic partial differential equation, with Dirichlet boundary conditions, and null initial condition:

$$\frac{\partial^{\beta}}{\partial t^{\beta}} c(t,\mathbf{x}) + (-\Delta_D)^{\alpha/2} (I - \Delta_D)^{\gamma/2} c(t,\mathbf{x}) = I_t^{1-\beta} \varepsilon(t,\mathbf{x}), \ \mathbf{x} \in D$$
$$c(t,\mathbf{x}) = 0, \quad \mathbf{x} \in \partial D, \quad \forall t, \quad c(0,\mathbf{x}) = 0, \quad \forall \mathbf{x} \in D \subset \mathbb{R}^n,$$

for $\beta \in (0,1)$, $\alpha + \gamma > n$, where equality is understood in the mean-square sense, and $-\Delta_D$ is the Dirichlet negative Laplacian operator on a regular bounded open domain $D \subset \mathbb{R}^n$. Here, the driven process, $I_t^{1-\beta} \varepsilon = \frac{1}{\Gamma(1-\beta)} \int_0^t (t-u)^{-\beta} \varepsilon(u) du$, is constructed as the Riemann–Liouville fractional integral of order $\beta - 1$ of a space-time zero-mean white noise ε (see, for example, Samko, Kilbas and Marichev, 1993). The local regularity properties, in the mean-square sense and in the sample-path sense, of the weak-sense solution in the fractional Sobolev space $\overline{H}^{\alpha+\gamma}(D)$, have been analyzed in Anh, Leonenko and Ruiz-Medina (2016), under the assumption of the Gaussian distribution of the driven process. The present paper investigates the distributional characteristics of the derived weak-sense solution beyond the Gaussian case.

This is a joint work with Vo Anh (Queensland University of Technology, Australia) and Nikolai Leonenko (Cardiff University, UK).

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[2] Anh, V.V., Leonenko, N.N. and Ruiz-Medina, M.D. (2016), Space-time fractional stochastic equations on regular bounded open domains. (Submitted).

Gennady Samorodnitsky (Cornell University, USA)

New classes of self-similar stable processes with stationary increments and functional central limit theorems for heavy tailed stationary infinitely divisible processes generated by conservative flows

We introduce an entirely new class of stable self-similar processes with stationary increments. Special cases of these processes arise as limits in functional central limit theorems for partial sums of symmetric stationary long-range dependent heavy tailed infinitely divisible processes. The infinitely

[1] M. D. Ruiz-Medina is supported by MTM2015–71839–P project of the DGI, MINECO

divisible process has long memory that generates a Mittag–Leffler process as an ingredient of the limit. The Mittag–Leffler process is induced by an associated Harris chain, at the discrete-time level. The infinitely divisible process also has certain negative dependence. The negative dependence is due to cancellations arising from Gaussian-type fluctuations of functionals of the associated Harris chain. The new types of limiting processes involve stable random measures, due to heavy tails, Mittag–Leffler processes, due to long memory, and Brownian motions, due to the Gaussian second order cancellations.

This talk is based on joint work with Paul Jung (University of Alabama Birmingham, USA) and Takashi Owada (Technion, Israel).

Georgiy Shevchenko (Taras Shevchenko National University of Kyiv)

Extended fractional integral and representation results for fractional Brownian motion

The talk will be devoted to stochastic integral representations of the form

$$\xi = \int_0^T \psi(s) dB^H(s),$$

where ϕ is an adapted process, and B^H is a fractional Brownian motion with Hurst parameter $H > 1/2$. Studying such representations is motived by applications in financial mathematics, where ϕ plays a role of a risky component of self-financing portfolio.

It will be discussed how the estimates for probabilities of small deviations of realized quadratic variation of B^H can be used to construct integral representations of the form under some mild assumptions on ξ. To this end, we define a generalization of fractional integral, introduced by Martina Zähle.

The talk is based on a joint research with Taras Shalaiko (Mannheim University, Germany).

Tommi Sottinen (University of Vaasa, Finland)

Parameter Estimation for the Langevin Equation with Stationary-Increment Gaussian Noise [1]

We study the Langevin equation with stationary-increment Gaussian noise. We show the strong consistency and the asymptotic normality with Berry–Esseen bound of the so-called alternative estimator of the mean reversion parameter. The strong consistency follows from the ergodicity of the stationary solution, while the asymptotic normality and Berry-Esseen bound follow from the fourth moment theorem. The conditions and results are stated in terms of the variance function of the noise. We consider both the case of continuous and discrete observations. As examples we consider fractional and bifractional Ornstein–Uhlenbeck processes. Finally, we discuss the maximum likelihood and the least squares estimators.

This talk is based on joint work with Lauri Viitasaari (Aalto University, Finland).

[1] Sottinen, T., Viitasaari, L. *Parameter Estimation for the Langevin Equation with Stationary-Increment Gaussian Noise*. Preprint, arXiv:1603.00390, 2016, 37 pages

Murad S. Taqqu (Boston University, USA)

Behavior of the generalized Rosenblatt process at extreme critical exponent values

The generalized Rosenblatt process is obtained by replacing the single critical exponent characterizing the Rosenblatt process by two different exponents living in the interior of a triangular region. What happens to that generalized Rosenblatt process as these critical exponents approach

[1] T. Sottinen was partially funded by the Finnish Cultural Foundation (National Foundations' Professor Pool)

the boundaries of the triangle? We show that on each of the two symmetric boundaries, the limit is non-Gaussian. On the third boundary, the limit is Brownian motion. The rates of convergence to these boundaries are also given. The situation is particularly delicate as one approaches the corners of the triangle, because the limit process will depend on how these corners are approached. All limits are in the sense of weak convergence in $C[0,1]$.

This is joint work with Shuyang Bai.

Bruno Toaldo (Sapienza Università di Roma)

Semi-Markov processes and integro-differential equations

A stochastic process X is semi-Markov in the sense of Gihman and Skorohod if the couple $(X(t), J(t))$, where $J(t)$ is the sojourn time in the current position, is a strict Markov process. The waiting times between the changes of state of the process are in general non exponential r.v.'s. We study the governing equations (Kolmogorov type equations) for semi-Markov processes, by distinguishing between stepped and non-stepped processes. It turns out that these equations are integro-differential equations that become fractional in some particular and interesting cases. For example this is true for stepped processes when the distribution of the waiting times are i.i.d. r.v.'s J_i satisfying $P(J_i > t) = E_\alpha(-t^\alpha)$, $\alpha \in (0,1)$, where $E_\alpha(\cdot)$ is the Mittag–Leffler function. Our method is based on Markov embedding since this permits to use some theory of renewal processes and Lévy processes.

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[3] V.N. Kolokoltsov. *Generalized Continuous-Time Random Walks, subordination by hitting times, and fractional dynamics*. Theory Probab. Appl., volume 53, 2009, pages 594–609.

[4] M.M. Meerschaert and B. Toaldo, *Relaxation patterns and semi-Markov dynamics.* In revision.

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[7] B. Toaldo, *Convolution-type derivatives, hitting-times of subordinators and time-changed C_0-semigroups.* Pot. Anal., volume 42, number 1, 2015, pages 115 – 140.

Grygoriy M. Torbin (National Dragomanov University, Ukraine)

On fractal dimensions faithfulness and its applications

Let $\dim_H(E,\Phi)$ resp. $\dim_P(E,\Phi)$ be the Hausdorff resp. packing dimension of a set E w.r.t. a Vitaly coverings Φ. The family Φ is said to be a *faithful* for the Hausdorff resp. packing dimension calculation on a metric space (M,ρ) if $\dim_H(E,\Phi) = \dim_H(E), \quad \forall E \subseteq M$ resp. $\dim_P(E,\Phi) = \dim_P(E), \quad \forall E \subseteq M$.

During the talk we'll present new techniques to prove faithfulness/non-faithfulness for the family of cylinders generated by different expansions of real numbers and apply it to get new necessary and sufficient conditions for the faithfulness of cylinders generated by different expansions of real numbers over finite as well as infinite alphabets.

Connections between faithfulness of net coverings and the theories of DP-transformations resp. PDP-transformations (i.e., transformations preserving the Hausdorff resp. packing dimension) will also be discussed.

We also study problems of the faithfulness for the enlarged family $\hat{\Phi}$ of hypercylinders generated by different expansions and develop related theory of transformations preserving the Lebesgue structure and fine fractal properties of probability distributions. Discussion on possible applications of the results in metric and dimensional theories of non-normal numbers are also planned.

The talk is based on joint research with S. Albeverio, I. Garko, M. Ibragim, Yu. Kondratiev, M. Lebid, R. Nikiforov, Yu. Peres, O. Slutskyi.

[1] Albeverio, S., Garko, I., Ibragim, M., Torbin, G., *Non-normal numbers: Full Hausdorff dimensionality vs zero dimensionality*. accepted for publication in Bull. Sci.Math. (2016), http://www.sciencedirect.com/science/article/pii/S0007449716300380

[2] Albeverio, S., Kondratiev, Yu., Nikiforov, R., Torbin, G., *On the fractal phenomena connected with infinite linear IFS.* submitted to Math. Nachr., http://arxiv.org/abs/1507.05672.pdf

[3] Kondratiev, Yu., Lebid M., Slutskyi O., Torbin, G., *Cantor series expansions and packing dimension faithfulness.* submitted to Adv.Math., http://arxiv.org/abs/1507.05663

Ciprian A. Tudor (Université de Lille 1, France)

Fractional processes and the solution to the heat equation

The solutions to the heat equations with time-space white noise (but also with other correlated noises) are Gaussian fields which are strongly connected with certain fractional processes (fractional Brownian motion, bifractional Brownian motion). We will explain this connection and we will also discuss some consequences to the sample path properties or to the quadratic variations of the solution to the heat equation.

Martina Zähle (Friedrich Schiller University of Jena, Germany)

Ordinary and partial differential equations with fractional noise

We give a survey on recent developments in applications of fractional calculus to ODE and PDE with noises generated by (random) mappings of fractional smoothness ≥ 1/2. For Euclidean vector valued SDE some extensions of earlier results are presented. We further consider parabolic PDE with spatial arguments in certain metric measure spaces. Some fractal models are included. The second order (pseudo)differential operator is given by means of an associated Markovian semigroup and the regularity of the solution is measured in terms of fractional Bessel potential spaces and Hölder spaces. For making the noise precise duality arguments in space and fractional derivatives in time are used. Finally, we consider Gaussian space-time fields in arbitrary D-regular spaces with upper variance estimates as for Euclidean fractional Brownian sheets. We show the existence of strong Hölder continuous modifications. This leads to examples for the above noises.

Mikhail Zhitlukhin (Steklov Institute of Mathematics)

Bounds for the expected maximum of a fractional Brownian motion [1]

We establish upper and lower bounds for $\mathrm{E}\max_{0\le t\le 1} B_t^H$ for a fractional Brownian motion B^H and investigate the rate of convergence to that quantity of its discrete approximation $\mathrm{E}\max_{0\le i\le n} B_{i/n}^H$. The main result shows that the convergence rate can be estimated from above by $C\sqrt{\ln n}/n^H$ with a constant C not depending on n and H. We also establish some further properties of these two maxima as functions of n and H. The results generalize to a class of Gaussian processes with covariance function similar to that of a fractional Brownian motion.

This is a joint work with K. Borovkov, Yu. Mishura, A. Novikov.

[1] Borovkov, K., Mishura, Yu., Novikov, A., Zhitlukhin M. *Bounds for expected maxima of Gaussian processes and their discrete approximations*, Stochastics, published online 30 Dec 2015.

[1] Mikhail Zhitlukhin is supported by the Russian Science Foundation under grant 14-21-00162.

Mounir Zili (University of Monastir, Tunisia)

Mixed sub-fractional Brownian motion

The sub-fractional Brownian motion is an extension of a Brownian motion, which was investigated in many papers (e.g. [1], [3]). It is a stochastic process $\xi^H = \left(\xi_t^H\right)_{t\geq 0}$, defined on a probability space $(\Omega, F, \mathbb{P})$ by:

$$\xi_t^H := \frac{B_t^H + B_{-t}^H}{\sqrt{2}},$$

where $H \in (0,1)$ and $\left(B_t^H\right)_{t\in\mathbb{R}}$ is a fractional Brownian motion on the whole real line. In this communication, we introduce an extension of the sub-fractional Brownian motion, which could serve to get a good model of certain phenomena, taking not only the sign, as in the case of the sub fractional Brownian motion, but also the strength of dependence between the increments of the phenomena into account. We present some basic properties of this process, its non-Markovian and non-stationarity characteristics, the conditions under which it is a semimartingale, and some fractal features of its sample paths. Especially we investigate the Hausdorff dimensions of its graph, range and level sets.

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